Review: Properties of Logarithms & Exponents 5/22/14

Logarithmic functions are the *inverse* of exponential functions.

Exponential functions:

Domain: all real numbers

Range: $y > 0$

Horizontal asymptote: $y = 0$

Logarithmic functions:

Domain: $x > 0$

Range: all real numbers

Vertical asymptote: $x = 0$

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exponential function

$$f(x) = a * b^x$$

$$a = \text{initial amount}$$
$$b = \text{growth factor}$$

common logarithm

$$\log_{10} x = \log x$$

natural logarithm

$$y = \ln x \qquad x = e^y$$

$$y = \log_e x$$

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Power Property

$$\log(a^c) = c\log(a)$$

Change of base property

$$\log_b a = \frac{\log a}{\log b}$$

Plotting the graph

$$\log_b x$$

b > 1 increasing

0 < b < 1 decreasing

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Multiplication Law of Exponents & Logarithms

$$\log(a * b) = \log(a) + \log(b)$$

$$a^m * a^n = a^{m+n}$$

Subtraction Law of Exponents & Logarithms

$$\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$$

$$\frac{a^m}{a^n} = a^{m-n}$$

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Zero Property

$$\log_b 1 = 0$$

$$b^0 = 1$$

Identity Property

$$\log_b b = 1$$

$$b^1 = b$$

Equivalent Exponent Forms

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Fraction to a Power

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

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Solve:

$$\log\left(4x^2\right) + 2\log\left(\frac{1}{x}\right) =$$

$$\log_2\left(\frac{1}{16}\right) =$$

$$\frac{1}{5}\log\left(\frac{1}{x^2}\right) = 1$$

Simple Interest

$$P_T = P_0 + \underbrace{P_0 r t}_{\text{Interest}}$$

Compound Interest

$$P_T = P_0\left(1+\frac{r}{n}\right)^{nt}$$

n = compounding per year

Arithmetic & Geometric Sequences & Series

Summation notation $\sum_{i=1}^{n} f(x)$

Expand

$$\sum_{i=1}^{n} f(x) = f(1) + f(2) + \ldots + f(n)$$

Expand and evaluate

$$\sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5)$$

$$= 2 + 4 + 6 + 8 + 10 = 30$$

Arithmetic & Geometric Sequences & Series

Arithmetic Sequences & Series

Explicit formula: $a_n = a_1 + (n-1) * d$

Simple formula: $a_n = d * n + (a_1 - d)$

Recursive formula: $a_n = a_{n-1} + d$

Finite series: $S_n = \frac{n}{2}(a_1 + a_n)$

$$n = \frac{(a_n - a_1)}{d} + 1$$

Arithmetic & Geometric Sequences & Series

Geometric Sequences & Series

Explicit formula: $a_n = a_1 * r^{n-1}$

Recursive formula: $a_n = r * a_{n-1}$

Finite series: $S_n = \frac{a_1(1-r^n)}{1-r}$

$$n = \frac{\log\left(\frac{a_n}{a_1}\right)}{\log(r)} + 1$$

Infinite convergent series: $S_\infty = \frac{a_1}{1-r}$

Arithmetic & Geometric Sequences & Series

Solve:

Find the 100th term in this sequence:
123, 116, 109, ...

Find the sums of the following series:
-15 -3 +9 +21 ... +93

$$\sum_{i=1}^{10} 5 * 2i$$

$$\sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^i$$

Rational Expressions & Equations

- Rational expression: a ratio containing a variable
- Proportion: an equality which compares two ratios or rational expressions
- A rational expression is <u>equal to zero</u> if the numerator is equal to zero.
- A rational expression is <u>undefined</u> if the denominator is equal to zero.
- A rational expression may be simplified if it is not undefined.

Rational Expressions & Equations

- Rational expressions are multiplied, divided, added, and subtracted in the same ways as fractions without variables.
- Direct variation: as x increases, y increases
- Inverse variation: as x increases, y decreases
- Vertical asymptotes exist for the values of x which cause a rational expression to be undefined.
- There is one vertical asymptote for every different x term in the denominator.

Radical Expressions & Equations

If a and b are factors of c, then $\sqrt{c} = \sqrt{a * b}$.

Also, $\sqrt{a * b} = \sqrt{a} * \sqrt{b}$.

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}} = a^{\frac{m}{n}}$$

$$a^{m.np} = a^m + a^{\frac{n}{10}} + a^{\frac{p}{100}}$$

Radical Expressions & Equations

- You may not take the square root of a negative number and have a real root.
- You may add or subtract radical expressions only if the radicand of each term is the same.
- When multiplying radical expressions, coefficients multiply by coefficients and radicands multiply by radicands.

Recall $\sqrt{-1} = i$ is an imaginary number, and that

$$\sqrt{-36} = \sqrt{-1 * 36} = \sqrt{36} * \sqrt{-1} = 6i$$

therefore $6i$ is an imaginary number.

Radical Expressions & Equations

- Complex numbers are written a+bi.
- Adding complex numbers:

  (a+bi) + (c+di) = (a+c) + (bi + di)
- Multiplying complex numbers:

  (a+bi)(c+di) = ac + ad$i$ + bc$i$ + bdi^2

  Substitute (-1) for i^2 and combine like terms.
- Dividing complex numbers: Rationalize the denominator by multiplying by its conjugate as a form of one.

$$\frac{a+bi}{c+di} * \frac{c-di}{c-di} = \frac{ac-adi+bci-bdi^2}{c^2-d^2i^2} = \frac{\left(ac+bdi^2\right)+\left(bci-adi\right)}{c^2+d^2}$$